

# First Sperlonga Summer School on Mechanics and Engineering Sciences

This is the first edition of the Sperlonga Summer School on Mechanics and Engineering Sciences, run jointly by the Fondazione “Tullio Levi-Civita” of Cisterna di Latina and the Italian Association of Theoretical and Applied Mechanics (AIMETA), with the support of the town of Sperlonga – the permanent seat of the School.

Courses and seminars, organized with an interdisciplinary attitude, aim at introducing young scientists to present-day developments in mechanics at the interface with mathematics, physics, materials science, biology, and engineering.

Lectures will be complemented by discussion sessions, to foster lively interactions among participants.

The School is mainly addressed to young researchers and doctoral students in mathematics, physics and engineering.

## G. Del Piero (Università di Ferrara, Italy)

### ***A variational approach to fracture and other inelastic phenomena***

There is currently a large interest in the variational approach to fracture and in its extension outside the domain of fracture mechanics. The aim of this Course is to provide an outlook on the trends in the subject, with emphasis on the interrelation between the mechanical modeling of material response and the theoretical foundations of Continuum Mechanics.

The modeling aspect will be centered on the “cohesive energy” model developed in [1]. It is based on the minimization of an energy functional given by the sum of two terms: an elastic strain energy associated with the bulk deformation of the continuum, and a cohesive energy concentrated on singular discontinuity surfaces of the displacement field. It will be shown that the material response is strongly conditioned by the shape of the cohesive energy function in the vicinity of the origin. The cohesive energy approach allows to describe with a unique model such different types of material responses as fracture and plasticity. The insufficiency of global minimization will be demonstrated, by showing that there is experimental evidence that the quasi-static evolution processes are made of local minimizers.

The foundational aspects will deal with the dissipative character of the cohesive energy, and with some incremental minimization techniques in the presence of dissipative phenomena, developed by Mielke [2] and co-authors. The course will be completed by a short account of some recent developments based on the definition of a diffuse cohesive energy, as opposed to the energy concentrated on singular surfaces.

[1] G. Del Piero, L. Truskinovsky, *Elastic bars with cohesive energy*, Cont. Mech. Thermodyn. 21: 141-171, 2009

[2] A. Mielke, *Modeling and analysis of rate-independent processes*, Lipschitz Lecture, Bonn 2007

## E. Presutti (Università di Roma Tor Vergata, Italy)

### ***Macroscopic limits in statistical mechanics***

A same substance made of same molecules mutually interacting with same forces may nonetheless appear to a macroscopic observation in dramatically different states: just think of ice, water and vapor which are different phases of a same system (of  $H_2O$ )

molecules). Thus something very singular must happen in passing from microscopic to macroscopic, as the phase transition mentioned above.

Part I. We shall start by discussing the issue at 0 temperature, where the equilibrium states are defined by minimizing the energy (ground states). We thus consider a system of point particles enclosed in some box which are mutually interacting, their interactions are described by a hamiltonian and the equilibria are the minimizers of the hamiltonian. Macroscopic behavior describes the structure of the minimizers when the box gets larger at fixed density, ideal macroscopic behavior being reached in the macroscopic limit where the box invades the whole space and the number of particles becomes then infinite. Realistic interactions among molecules are described by Lennard-Jones potentials and it has been proved, see [2], that for such potentials a phase transition occurs in the macroscopic limit from a vapor to a solid phase.

Part II. There is no proof of the analogous phenomenon when the temperature is raised. Equilibrium at non zero temperatures is no longer obtained by minimizing the energy as we must take into account as well the "number of microscopic states" which have the same energy: the balance between the two, weighted by the temperature, is known in thermodynamics to define the equilibrium. We shall follow Boltzmann to translate these ideas into a definition of thermodynamic equilibrium states [Gibbs states]. This involves the notion of "entropy" and establishes interesting connections with information theory, probability and dynamical systems theory.

Part III. The analysis of the Gibbs states at non zero temperatures is very complex and we still lack a proof that for realistic interactions the equilibrium states are described by the typical phase diagrams which appear in thermodynamics. Much more is known if we study Kac potentials, namely if we suppose a sharp separation of scales with the range of the interaction much larger than the typical interparticle distances. Kac proposal in the 60s was to use such ideas to derive the van der Waals theory of liquid-vapor phase transitions. An important step in this project was achieved by Lebowitz and Penrose who proved that in a mesoscopic limit (intermediate between microscopic and macroscopic) the equilibrium is obtained by minimizing a free energy functional where both energy and entropy appear weighted by the temperature. In this limit therefore we are, as at 0 temperature, reduced to a variational problem with the energy replaced by the free energy functional.

Part IV. The Lebowitz-Penrose functionals are non local functionals which are analogous to the Ginzburg-Landau functionals studied in continuum theories, the gradient penalization present in the latter being replaced by the non local interaction in the former. From a technical viewpoint this makes a big change as the Lebowitz-Penrose functionals do not have the coercivity property (lack of compactness) of the Ginzburg-Landau functionals. The problem of coexistence of phases for the Lebowitz-Penrose functionals in the macroscopic limit can be reduced to a problem of Gamma-convergence which, despite the absence of coercivity, can be carried through. We shall discuss these issues together with the implications of the theory on the behavior of the underlying particle system with a brief sketch of the proof that the liquid-vapor phase transition for the functional can be translated into a proof for the particles as well.

For Part II, III, IV we refer to [1] and references therein, for Part I to [2].

[1] E. Presutti. Scaling Limits in Statistical Mechanics and Microstructures in Continuum Mechanics. HEIDELBERG: Springer, (2008).

[2] F. Theil. A proof of crystallization in two dimensions. Commun. Math. Phys. 262, 209-236, 2006.

In addition two regular lectures, the following topical seminars will be held:

**Jean-Jacques Marigo (Universite' Pierre et Marie Curie, Paris)**

**1. Construction of fatigue laws from cohesive force models: the 2D case**

**2. Dynamical effects in the propagation of cracks due to a toughness**

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or visit

[www.fondazionelevicivita.it/sperlonga2011](http://www.fondazionelevicivita.it/sperlonga2011)